

# Quantum Pasts and the Utility of History\*

James B. Hartle<sup>†</sup>

*Institute of Theoretical Physics  
and Department of Physics,  
University of California,  
Santa Barbara, CA 93106-4030*

## Abstract

From data in the present we can predict the future and retrodict the past. These predictions and retrodictions are for histories — most simply time sequences of events. Quantum mechanics gives probabilities for individual histories in a decoherent set of alternative histories. This paper discusses several issues connected with the distinction between prediction and retrodiction in quantum cosmology: the difference between classical and quantum retrodiction, the permanence of the past, why we predict the future but remember the past, the nature and utility of reconstructing the past(s), and information theoretic measures of the utility of history.

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<sup>†</sup>e-mail: hartle@cosmic.physics.ucsb.edu

## I. INTRODUCTION

Up here on length scales above the Planck length, where the notion of an approximately classical spacetime makes sense, the world is four-dimensional with three spacelike dimensions and one for time. Classically we most accurately describe the world in terms of the four-dimensional concepts of points, world lines, and field configurations in spacetime. Yet we can divide our subjective experience up into past, present, and future. From our point of view these have very different properties: We know the present, remember the past, and predict the future.

The processes of predicting the future and retrodicting the past are familiar and elementary in classical physics. However, fundamentally the world is not classical but quantum mechanical. The process of prediction — the prediction of a measurement outcome, for instance — is also familiar in quantum mechanics. The quantum past, however is more of a foreign country. This essay is concerned with the past in quantum mechanics. What is the nature of quantum mechanical retrodiction? What is the origin of the differences between past and future? Most importantly, why are we interested in the past at all? It's over. What is the utility of history?

## II. PREDICTION AND RETRODICTION IN QUANTUM MECHANICS

### A. Classical Prediction and Retrodiction

Classically, prediction and retrodiction are symmetrically related if we presume that the fundamental equations of motion are time reversal invariant. From present values for the positions and velocities of the molecules in a gas we may predict their future positions and velocities and retrodict their past ones by using the deterministic equations of motion. Effective classical equations for coarse-grained quantities such as the density of the gas may exhibit dissipation, chaos, and history dependence which distinguish prediction from retrodiction, but *in principle*, if present data contained enough information, the processes of prediction and retrodiction would be symmetrically related. That is not the case in quantum mechanics.

### B. The Quantum Mechanics of a Closed System

We consider the quantum processes of prediction and retrodiction from the most comprehensive perspective — the quantum mechanics of a closed system, most generally the universe as a whole. (See, *e.g.* [1]). We neglect quantum gravity to keep the discussion manageable. This is an excellent approximation for many useful purposes for any time later than  $10^{-43}$ sec after the big bang. In this approximation the universe may be thought of as a collection of matter fields inside a large, perhaps expanding, box in fixed background spacetime geometry. The two fundamental inputs that specify a closed quantum mechanical system are its Hamiltonian  $H$  and initial quantum state  $|\Psi\rangle$ . We now very briefly review how these lead to predictions and retrodictions.

General objectives of quantum theory are the probabilities of individual time histories in sets of alternative coarse-grained histories of the universe. Examples are the probabilities of a set of possible orbits of the earth around the sun. Such histories are said to be coarse grained because they do not specify the coördinates of every particle in the universe but only those of the center of mass of the sun and earth and these only crudely and not at every time.

The simplest sets of alternative histories are alternatives at a moment of time. These can always be reduced to a set of yes/no alternatives. For instance, questions about the position of the earth's center of mass can be reduced to questions of the form: "Is the earth's center of mass in this region — yes or no?", "Is it in that region — yes or no?", *etc.* A set of yes/no alternatives at a moment of time, say  $t = 0$ , is represented by a set of orthogonal projection operators  $\{P_\alpha\}$ ,  $\alpha = 1, 2, \dots$  — one projection operator for each alternative. These projection operators must satisfy

$$\sum_\alpha P_\alpha = I, \quad \text{and} \quad P_\alpha P_\beta = 0, \quad \alpha \neq \beta, \quad (2.1)$$

showing that they represent an exhaustive set of exclusive alternatives. In the Heisenberg picture that we shall employ, the same set of alternatives at a later time is represented by a different set of operators related to the first by

$$P_\alpha(t) = e^{iHt} P_\alpha e^{-iHt}. \quad (2.2)$$

(Here and throughout we use units where  $\hbar = 1$ .)

A set of alternative histories may be specified by giving sets of alternatives at a series of times  $t_1 < t_2 < \dots < t_n$ . We denote the corresponding sets of projections by  $\{P_{\alpha_1}^1(t_1)\}, \dots, \{P_{\alpha_n}^n(t_n)\}$ . An individual history corresponds to a particular sequence of alternatives  $(\alpha_1, \dots, \alpha_n) \equiv \alpha$  and is represented by the corresponding chain of projections

$$C_\alpha = P_{\alpha_n}^n(t_n) \cdots P_{\alpha_1}^1(t_1). \quad (2.3)$$

Such a set of histories is said to be *coarse-grained* when there is not a set of alternatives at each and every time or when the projections are not one-dimensional onto complete sets of states. For a set of histories describing alternative orbits of the earth, the  $\{P_{\alpha_k}^k(t_k)\}$  might be sets of projections onto an exhaustive set of ranges of position of the earth's center of mass at different times  $t_k$ . A coarse-grained individual orbit is a sequence of these ranges at a series of times.

When the  $C_\alpha$  act on the initial state they give branch state vectors  $C_\alpha|\Psi\rangle$  for each of the alternative histories in the set. The probabilities of the individual histories are

$$p(\alpha) = \|C_\alpha|\Psi\rangle\|^2. \quad (2.4)$$

Eq.(2.4) displays an important asymmetry in time. There is  $|\Psi\rangle$  on one end of history and nothing on the other. That is the arrow of time in quantum theory. This is the only fundamental time asymmetry and all other "arrows of time" must arise from it and particular properties of the  $|\Psi\rangle$  of our universe. Similarly this is the only source for the distinction between past and future. The question is not "Where does the distinction between past and future come from?" but rather "How does the distinction between past and future arise from

special properties of the initial condition of the universe in conjunction with the quantum mechanical arrow of time?”

However, it is a convention that the state  $|\Psi\rangle$  is an *initial* condition at a moment earlier than all others in history. Field theory is *CPT* invariant, and by making use of a *CPT* transformation the time order of the the operators and  $|\Psi\rangle$  could be inverted so that  $|\Psi\rangle$  was a final condition. That convention however is in conflict with ordinary language and therefore risks confusion. Thus in this paper, by “past” we mean the times on the side of the present which is closest to the initial condition.

Quantum mechanics can be formulated time neutrally with initial and final density matrices  $\rho^i$  and  $\rho^f$  entering symmetrically into the expressions for probabilities [2]. So formulated, quantum mechanics contains no intrinsic direction of time, and all arrows of time arise from asymmetries between the initial and final conditions. It appears that a final condition of indifference,  $\rho^f = I/Tr(I)$  and a special initial condition, possibly pure,  $\rho^i = |\Psi\rangle\langle\Psi|$ , are consistent with known data [3,4]. In that case the time neutral formulation reduces to the one used here summarized by (2.4).

Probabilities cannot be consistently assigned to *every* set of alternative histories of a closed system because of quantum mechanical interference. In the two-slit experiment, an electron emitted by an electron gun can pass through either of two slits in a screen on its way to detection at a further screen. It would be inconsistent to assign probabilities to the two histories distinguished by which slit the electron goes through. The probability to arrive at a definite point on the detecting screen would not be the sum of the probabilities to arrive there by going through each of the two different slits because of interference. The sets of histories to which probabilities *can* be assigned are those which have negligible interference between their individual members, that is

$$\langle\Psi|C_\alpha^\dagger C_\beta|\Psi\rangle \approx 0, \alpha \neq \beta. \quad (2.5)$$

Such sets of histories are said to *decohere* and (2.5) is a decoherence condition. The probabilities (2.4) satisfy the additivity properties required of probabilities as a result of (2.5). As a consequence (2.4) is a consistent assignment of probabilities for decoherent sets. The sets of alternative histories which can be assigned probabilities are thus determined through (2.5) by the initial condition of the universe.

As an example of a mechanism of decoherence, think of a millimeter-size dust grain in a superposition of two positions deep in intergalactic space [5]. Consider alternative histories of the position of this particle at a series of a few times. Were the grain truly isolated this situation would be analogous to the two slit experiment and alternative histories of its position would not decohere. However, in our universe, even deep in space, about  $10^{11}$  cosmic blackbody photons scatter from it every second. Through these interactions, states of the seemingly isolated dust grain become strongly correlated with states of the radiation. Two states of position differing by a millimeter become correlated with nearly orthogonal states of the radiation after about a nanosecond. The branch state vectors corresponding to different suitably coarse-grained histories of position thereby become nearly orthogonal, eq. (2.5) is satisfied, and decoherence achieved. Mechanisms such as this are widespread in our universe and cause the habitual decoherence of the kinds of quasiclassical variable we so often find it useful to follow. In what follows we shall always assume that we are dealing with a decoherent set of histories.

### C. Prediction and Retrodiction

From known present data we aim to predict the future and retrodict the past. The exact nature of this present data is not important for the subsequent discussion. It could be the memories of an individual observer, the records of a collection of them, or an alternative having nothing to do with observers such as the present size of the universe. Neither does the data from which predictions and retrodictions are inferred have to be at one moment of time, and indeed typically it is not. However, to simplify the exposition, we shall assume that it is; the generalization to data at several times should be clear.

Probabilities of future alternatives conditioned on present data are relevant for prediction. Probabilities for past alternatives conditioned on the same present data are required for retrodiction. These are easily constructed from the joint probabilities (2.4) of histories that include present data, as we now show.

Let  $t_0$  be the time of the present and denote the projection corresponding to present data by  $P_{pd(t_0)}(t_0)$ . The awkward subscript  $pd(t)$  is necessary because present data will change in time as new data is acquired and old data lost. The operator  $P_{pd(t_0)}(t)$ , evolved from  $t_0$  according to (2.4), would represent that same alternative as  $P_{pd(t_0)}(t_0)$  at time  $t$ , but that is not present data at time  $t$ . However, since we only use the operator  $P_{pd(t_0)}(t_0)$ , we shall often abbreviate it as  $P_{pd(t_0)}$  or even  $P_{pd}$ .

The conditional probability for a chain of alternatives  $(\alpha_n, \dots, \alpha_1)$  at times  $t_n > \dots > t_1$  all to the future of  $t_0$  is

$$p(\alpha_n t_n, \dots, \alpha_1 t_1 | pd t_0) = \frac{p(\alpha_n t_n, \dots, \alpha_1 t_1, pd t_0)}{p(pd t_0)} . \quad (2.6)$$

Using (2.4), and assuming the set of histories is decoherent, the joint probabilities in the numerator and denominator may be evaluated from the relevant chains of projections and the state  $|\Psi\rangle$ . The results are conditional probabilities for prediction.

Similarly for retrodiction. The conditional probabilities for a history of alternatives  $(\alpha_1, \dots, \alpha_n)$  at times  $\tau_1 > \dots > \tau_n$  all *prior* to  $t_0$  is

$$p(\alpha_1 \tau_1, \dots, \alpha_n \tau_n | pd t_0) = \frac{p(\alpha_1 \tau_1, \dots, \alpha_n \tau_n, pd t_0)}{p(pd t_0)} , \quad (2.7)$$

where the chain relevant for computing the numerator is

$$P_{pd(t_0)}(t_0) P_{\alpha_1}^1(\tau_1) \dots P_{\alpha_n}^n(\tau_n) . \quad (2.8)$$

Eq. (2.7) gives conditional probabilities for retrodiction.

For example, to predict or retrodict the orbit of the earth about the sun  $P_{pd(t_0)}$  might be the projection on a present record of previous observations of the earth's position to certain accuracies, and  $P_{\alpha_k}^k(t_k)$ ,  $k = 1, \dots, n$  might be projections on a series of future or past positions to given accuracies. For typical accuracies to which the position of the earth's center of mass is determined, we expect there to be a conditional probability near unity for classical orbits consistent with present data.

#### D. Retrodiction in the Approximate Quantum Mechanics of Measured Subsystems.

The Copenhagen formulation of quantum mechanics is an approximation to the more general quantum mechanics of closed systems. It is appropriate when dealing with measurement situations in which a subsystem is approximately isolated from the rest of the universe and evolves independently except for occasional interactions with a measuring apparatus<sup>1</sup>. In ideal situations, the decoherence of the alternative histories of the registrations by the apparatus may be approximated as exact, and the probabilities for the outcomes of measurements calculated from a formula similar to (2.4) but *involving the Hilbert space of the measured subsystem alone*. The change in focus from the Hilbert space of the  $10^{80}$  particles in the visible universe to say, the Hilbert space of the few particles involved in a typical experiment is an approximate simplification of overwhelming practical advantage.

There is an obstacle to using the Copenhagen probabilities to retrodict the past of a subsystem. The output of the Copenhagen approximation are the probabilities of the outcomes of measurements. But, there is not enough information in the present state of a subsystem to fix what past measurements were performed on it and when they were performed. The influence of these measurements on the history of the subsystem's state is generally non-negligible. To retrodict, the information about the nature and times of measurements must either be supplied, or perhaps itself retrodicted from a separate classical physics governing the measuring apparatus.

In this practical sense it is not possible to retrodict the past from the present data in the Copenhagen formulation of quantum mechanics, except for those situations where evolution is negligibly perturbed by external measurements as can be arranged in classical physics. A similar obstacle exists for prediction. This appears less serious because as observers we can control the nature of time of *future* measurements whereas those in the past seem beyond our reach. True quantum mechanical retrodiction is only possible for a closed system.

### III. QUANTUM PASTS

The reconstruction of the past is most honestly viewed in the context of retrodiction from present records. We retrodict the date 55BC for the first Roman invasion of Britain on the basis of present textual records. We use present observations of the sun and moon to reconstruct their past trajectories. We use fossil records to estimate that the probability is high that dinosaurs roamed the earth 250 million years ago. We infer that matter and radiation were in thermal equilibrium at the beginning of the universe from the present values of the Hubble constant, mean mass density, and the temperature of the cosmic background radiation.

We have seen how past history can be reconstructed in quantum mechanics through retrospective probabilities conditioned on present data. We now describe a number of non-classical features of quantum retrodiction:

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<sup>1</sup> See *e.g.* [6], Section II.10.

### A. The Present State is not Enough to Retrodict the Past

The conditional probabilities for *predictions* can be obtained from a present state. Define

$$|\Psi_{\text{present}}\rangle = \frac{P_{pd(t_0)}|\Psi\rangle}{\|P_{pd(t_0)}|\Psi\rangle\|}. \quad (3.1)$$

The probabilities (2.6) for future alternatives conditioned on present data are easily seen to be expressible as

$$p(\alpha_n t_n, \dots, \alpha_1 t_1 | pd t_0) = \|P_{\alpha_n}^n(t_n) \cdots P_{\alpha_1}^1(t_1) |\Psi_{\text{present}}\rangle\|^2. \quad (3.2)$$

The vector (3.1) is the usual notion of present state of the system. In this Heisenberg picture it is constant from the time of the initial condition up to the present and then “reduced” by the action of the projection representing present data. This present state contains all the information necessary for predicting the future as (3.2) shows.

By contrast no such summary is possible for retrodiction. The probabilities (2.7) for alternatives conditioned on present data cannot be expressed in terms of the projections and  $|\Psi_{\text{present}}\rangle$  alone. They require the initial condition  $|\Psi\rangle$  in addition to present data  $P_{pd(t_0)}$ . Put differently, correct probabilities for the past cannot generally be constructed simply by running the Schrödinger equation backwards in time from the present state. A theory of the initial state is required as well. In this respect the quantum mechanical notion of state at a moment of time is very different from the corresponding classical notion. Almost all retrodictions therefore depend, at least to some degree<sup>2</sup>, on assumptions about the initial state  $|\Psi\rangle$ . We cannot ignore this dependence. Complete ignorance of the initial state would be represented by an initial density matrix  $\rho = I/\text{Tr}(I)$  where  $I$  is the unit operator. But that is an initial condition of infinite temperature — patently inconsistent with observations today.

### B. The Past is Not Unique

We naturally think the past defined by alternative prior histories of the usual quasi-classical variables we are adapted to observe — histories of alternative values of densities of energy, momentum, chemical species, averaged over small volumes, *etc.* However, in quantum mechanics there are many other mutually incompatible possible pasts.

A simple mathematical example may help to illustrate this point in a trivial way. Consider a set of projections  $\{Q_\alpha\}$  on a basis of orthogonal states that include  $|\Psi\rangle$ . The set of histories

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<sup>2</sup>The trivial exception is the probability one retrodiction of a history defined by repeating the operator  $P_{pd}$  over and over at a series of past times. That is a kind of retrodiction following from the determinism of the Schrödinger equation alone. However, even if  $P_{pd}$  represents a quasiclassical alternative at the present time it corresponds to nothing like that at past times. Rather, operators representing quasiclassical alternative in the past are related to quasiclassical alternatives in the present by the Heisenberg equations of motion, (2.2).

$$P_{pd(t_0)}(t_0)Q_{\alpha_1}(\tau_1)Q_{\alpha_2}(\tau_2)\cdots Q_{\alpha_n}(\tau_n) \quad (3.3)$$

is decoherent for any assignment of times  $\tau_1 > \tau_2 > \cdots > \tau_n$  (all earlier than  $t_0$ ). The conditional probabilities

$$p(\alpha_1\tau_1, \cdots, \alpha_n\tau_n | pd\,t_0) \quad (3.4)$$

retrodict a past which is nothing like the usual quasiclassical one. Indeed the only retrodiction with non-zero probability is for the history where the  $Q$ 's are  $|\Psi\rangle\langle\Psi|$  repeated at every time.

We can illustrate this using Schrödinger's cat. Restricting attention to the usual thought experiment, we may infer from present data showing the cat to be alive a conditional probability near unity that the atom did not decay in the past. However, the conditional probability is also unity for the past alternative  $|\phi\rangle\langle\phi|$  where  $|\phi\rangle$  is the state which is a superposition of states in which the cat is alive and dead.

Indeed an arbitrary number of possible pasts can be constructed mathematically as follows: Take the vectors  $P_{pd(t_0)}|\Psi\rangle$  and  $\bar{P}_{pd(t_0)}|\Psi\rangle$  where  $\bar{P}_{pd(t_0)} = I - P_{pd(t_0)}$  ("not the present data"). Find a set of mutually orthogonal vectors such that half of them add to  $P_{pd(t_0)}|\Psi\rangle$  and the remaining ones to  $\bar{P}_{pd(t_0)}|\Psi\rangle$ . There are a great many ways of doing this. Pick a time  $\tau_1$  prior to  $t_0$  and write these vectors as

$$P_{pd(t_0)} P_{\alpha_1}^1(\tau_1) |\Psi\rangle, \quad \bar{P}_{pd(t_0)} P_{\beta_1}^1(\tau_1) |\Psi\rangle. \quad (3.5)$$

where  $\alpha_1$  and  $\beta_1$  range over half the set of vectors. This can always be done because the projections  $P_{\alpha_1}^1(\tau_1)$  can always be found, and, in the Heisenberg picture, a projection operator projects on a range of some quantity at any time. Repeat the process to add further projections and thereby construct a decoherent set of histories of prior alternatives that includes present data. That is a past in quantum mechanics. There are a very large number of ways of interpolating such projection operators and therefore a large number of quantum pasts.<sup>3</sup>

Many of these possible quantum pasts are mutually incompatible. That is, two decoherent sets of histories needed used in retrodiction are not generally coarse grainings of a finer grained decoherent set of histories containing them both. The two pasts in the Schrödinger cat example above are instances.<sup>4</sup> One can draw past inferences using one set or the other set but not both [1,11].

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<sup>3</sup>Other references discussing the implications of the multiplicity of quantum pasts are [7,8].

<sup>4</sup>Even two alternatives represented by orthogonal projections can both be retrodicted with probability one in different quantum pasts for certain present data and initial and final conditions [9]. Were two such alternatives to occur in the same past they would be exclusive — if one has probability one, the other necessarily has probability zero. No logical inconsistency arises from two orthogonal alternatives having probability one in *different* pasts, because, with the relevant initial and final conditions and present data, there are no decoherent sets of histories containing both orthogonal projections from whose probabilities a contradiction could be drawn [10].



The description of multiple quantum pasts requires care in the use of ordinary language which is adapted to a single quasiclassical past. For example, we are accustomed to say that an event “happened” in the past if its probability conditioned on present data is near unity. That is a statement which requires the history consisting of present data and the past alternative to be a member of *some* decoherent set of histories, but it is independent of *which* decoherent set because the value of the probability of a history is the same, eq.(2.4), in all decoherent sets of which it is a member. If we retain this meaning of “happened” in quantum theory we must accept mutually incompatible events can happen in different quantum pasts.<sup>5</sup>

What are we to make of these different quantum pasts? It is important to stress that, in general, we do not expect these to be constructed from operators representing anything like the alternatives of the usual quasiclassical realm. Further, in general, we must expect these to be pasts with distributed probabilities — no one past history with a probability near one on the basis of present data and the initial condition. The possible pasts can thus be expected to differ greatly in their utility. We shall return to this below.

### C. A Past is Not Necessarily Permanent

As we move into the future present data changes. Individually we make more observations and acquire more information over time. We also forget or lose access to much. Collectively we expect information to increase although much is erased or lost such as Aristotle’s *Comedy*.

Thus in general we expect  $P_{pd(t)}$  to become increasingly fine-grained with  $t$ . The decoherence of any particular quantum past is thus at risk. A coarse graining of a decoherent set is again decoherent, but a fine graining is not necessarily decoherent. If a set of past histories no longer decoheres in the presence of new data, we lose the ability to make these retrodictions. It is not that the past alternatives become less certain; there are no probabilities at all. A past is therefore not necessarily permanent in quantum mechanics.

This loss of the past occurs even in familiar laboratory situations. Consider a Stern-Gerlach thought experiment in which a beam of atoms, each initially in a superposition of spin states, is separated into two beams by passing through an inhomogeneous magnetic field, and later these beams are recombined. Suppose the atoms are in narrow wave packets so that we can meaningfully speak of the time a single atom is at a particular position along the beam. When the atom is in the region of separated beams, the set of histories defined by alternative spin states decoheres. (Or indeed at other times since histories consisting of alternatives at a single time always decohere.) However, a history which includes, in addition to these alternatives, alternative values of the spin at a time when the atom is in the region where the beams have been recombined will not decohere. The phases between the histories have been recovered in that present data by recombining the beams.

As we shall now describe, for quasiclassical pasts, it is genuinely or practically impossible to find present data with which alternative past histories of values of the usual quasiclassical

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<sup>5</sup>A prescription for avoiding ambiguities of ordinary language is to turn all statements into statements about quantum mechanical probabilities which are the output of quantum mechanics.

variables fail to decohere. As long as we stick to such alternatives we may proceed into the future secure in the knowledge that the quasiclassical past is permanent.<sup>6</sup>

#### IV. THE QUASICLASSICAL PAST

Classical physics is an approximation to quantum mechanics that is appropriate for particular coarse grainings and particular initial conditions such as the one which characterizes our universe. The classical past is unique, permanent, and retrodictable from present data alone — features which are not general in quantum theory as we have discussed. We now briefly describe how these features are recovered in the classical approximation to quantum theory.

The familiar variables of classical physics include averages over suitable volumes of densities of approximately conserved quantities such as energy, momentum, chemical species, *etc.* When the volumes are sufficiently large, sets of alternative histories of such variables can decohere, and exhibit approximate patterns of classical, deterministic correlations over time. The success of classical physics in describing phenomena over wide ranges of time place and epoch suggests that the initial condition and Hamiltonian of our universe exhibit a *quasiclassical realm* — a decohering set of alternative histories of quasiclassical variables that is maximally refined consistent with the requirements of decoherence and approximate classical predictability. Such a quasiclassical realm would extend over most of space and much of the history of the universe, and is a feature of the universe not of our choice. As observers in the universe we utilize coarse grainings of this usual quasiclassical realm to describe everyday experience and particular experimental situations.

To restrict to past histories of quasiclassical variables is to beg the question of the uniqueness of the past. However, there is still a sense in which we may speak of a unique quasiclassical past — the past of the usual quasiclassical realm. Different observers making use of usual quasiclassical variables for their particular present data and retrodictions construct pasts which are mutually compatible if they are all employing coarse grainings of the usual quasiclassical realm.

It is an open question whether the universe exhibits essentially distinct quasiclassical realms of high predictability characterized by variables that are different from the usual quasiclassical ones [13]. If, however, the usual quasiclassical realm is essentially alone in its high level of predictability that would be another sense in which the usual quasiclassical past is unique. In the following, to avoid cumbersome expressions, when we refer “the quasiclassical realm” or “quasiclassical variables” we mean the usual quasiclassical realm and the usual quasiclassical variables unless otherwise noted.

In a restricted class of cases, the equations of motion summarizing the temporal regularities of the quasiclassical realm enable coarse grainings of the quasiclassical past to be inferred from present data alone without further knowledge of the initial condition. The history of the solar system, for instance, can be inferred from the present positions and velocities of its

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<sup>6</sup>The decoherence condition (2.5) can be strengthened to ensure the permanence of the past. An example is the notion of *strong decoherence* [12].

constituents. However, this kind of retrodiction from present data is possible only in certain circumstances. The effective deterministic equations of motion of the usual quasiclassical realm are generally dissipative, chaotic, and dependent on the past history of the system.<sup>7</sup> Dissipation and chaos generally mean that past evolution is unstable. Dependence on past history means that present data is not enough to retrodict. Further, thermal and quantum noise arising from coarse graining can cause deviations from classical predictability, both for prediction and retrodiction. Only when the effects of dissipation, chaos, history dependence, and noise are sufficiently small can we use equations of motion to retrodict the usual quasiclassical past from present data alone. Unlike classical physics where coarse grainings and effective equations arise only out of ignorance, coarse graining is inevitable in quantum mechanics. Coarse graining is necessary for decoherence, and for the quasiclassical realm, coarse graining beyond that is necessary to achieve the approximate predictability in the face of the noise that typical mechanisms of decoherence produce [14].

It is impossible or hopelessly impractical to recover the phase information in present data that would lead to a failure of decoherence of the usual quasiclassical past. We mentioned that, for an intergalactic dust grain initially in a superposition of positions, alternative histories of subsequent position decohered because of their interaction with cosmic background photons. If later data includes the states of all scattered photons, then the past histories of position of the dust grain cannot be retrodicted from it — the relevant set of histories of position will not decohere. Of course, it is hopelessly impractical to recover such data. Some  $10^{11}$  photons scatter off the dust grain every second. Indeed, if we did not prepare ahead of time to recover the information, it's too late in principle. It left earlier at the speed of light. If any of those photons went down a black hole they are similarly irretrievably lost.

In science fiction we might imagine that even now intelligent aliens could be poised to recover the phases between past histories that we retrodict from present data. From this data they will not be able to retrodict the same past as we do. We, however, are unlikely to survive the process of collecting this data to compare notes.

In the quasiclassical past one can say with the poet, “The Moving Finger writes; and having writ, Moves on: nor all thy Piety nor Wit Shall lure it back to cancel half a Line, Nor all Thy tears wash out a Word of it.” That is not necessarily the case for all quantum mechanical pasts.

## V. WHY ARE THERE MORE RECORDS OF THE PAST THAN THE FUTURE?

In the quasiclassical realm, present data contain many more records of the past than of the future. In this paper a “record” is not presupposed to be of the past. A record is an alternative in the present that is correlated with high probability with another alternative *in the past or future*. That is, present data contains a record of an alternative  $\alpha_0$  if  $p(\alpha_0, t | p_d, t_0) \approx 1$  irrespective of the relation between  $t$  and  $t_0$ . A recorded alternative is thus either a near certain prediction or retrodiction from present data. For example, the

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<sup>7</sup> They are not dependent on future history; classical causality follows from quantum mechanical causality [14].

configuration of ink read 55BC in many texts (and 55BCE in others) is correlated with the Roman invasion of Britain. Certain configurations of neurons are correlated with our past experiences. We can also have records of the future. A table of the eclipses of the moon in AD2010 is just as much a record of that time as a table of eclipses in AD1000 is of its. The present position and velocity of the earth in its orbit around the sun are a record of its future positions as well as of its past ones. However, the preponderance of records seem to be of the past. That time asymmetry, like all others, can only be explained by the particular features of the initial condition  $|\Psi\rangle$ . In this section, we shall attempt to identify these features. We confine our attention entirely to the usual quasiclassical realm.

The second law of thermodynamics is the reason that present data contains more records of the past than the future. The Jaynes construction may be used to associate an entropy with every decoherent set of alternatives in quantum mechanics.<sup>8</sup> For example, the entropy of the coarse-grained set of histories consisting of alternatives  $\{P_\alpha(t)\}$  at a single moment of time is:

$$S(\{\alpha\}, t) = - \sum_\alpha p(\alpha, t) \log p(\alpha, t) + \sum_\alpha p(\alpha, t) \log \text{Tr}[P_\alpha(t)] . \quad (5.1)$$

where  $p(\alpha, t) = ||P_\alpha(t)|\psi\rangle||^2$  are the probabilities determined from the initial condition. Now follow this entropy with time keeping the coarse-graining fixed. When the  $P$ 's are projections on ranges of *quasiclassical variables* this entropy is low initially for the initial condition of our universe and therefore has a general tendency to increase afterwards.<sup>9</sup> That is the second law of thermodynamics and the origin of the thermodynamic arrow of time. As Boltzmann put it [19], "The second law of thermodynamics can be proved from the [time-reversible] mechanical theory if one assumes that the present state of the universe... started to evolve from an improbable state."

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<sup>8</sup> If that is not familiar see *e.g.* [15]. By extending the original Jaynes discussion an entropy can be defined for sets of histories [16].

<sup>9</sup> While low, the entropy is not as low as it could be. At the time of decoupling, matter and radiation were in near thermal equilibrium with an entropy of about  $10^{80}k$  inside the region visible from today. By contrast, the geometry of the early universe is as ordered as it could be. As Penrose has stressed [17], "gravitational entropy" is small compared to the maximal value of  $10^{120}k$  it could have if all that matter were in a black hole. The second law is a consequence of this initial geometrical order and the attractive nature of gravity.

For the entropy of the initial state to be near zero with respect to quasiclassical coarse grainings, it would have to predict a single configuration of quasiclassical variables with probability near unity [*cf.* (5.1)]. The subsequent approximate deterministic evolution of that configuration means that it must necessarily encode all the complexity of the quasiclassical realm we see about us. Such a near zero entropy initial condition would therefore be incomprehensibly complex. Rather the initial condition of our universe seems to be as *simple* as possible in a crude, intuitive sense: Matter in near thermal equilibrium which is maximum entropy for it, and highly ordered geometry which is minimal entropy for it because of the attractive nature of gravity. Quantum mechanical initial conditions which incorporate this simplicity (*e.g.* [18]) predict distributed probabilities for histories of the quasiclassical realm which is why there are few initial records of the future.

Similar second law behavior is expected for the entropy of a set of alternatives  $\{P_\alpha(t)\}$  conditioned on present data. This is given analogously to (5.1) by

$$S(\{\alpha\}, t|pd, t_0) = - \sum_{\alpha} p(\alpha, t|pd, t_0) \log p(\alpha, t|pd, t_0) \\ + \sum_{\alpha} p(\alpha, t|pd, t_0) \log \text{Tr}[P_\alpha(t)] . \quad (5.2)$$

where the conditional probabilities  $p(\alpha, t|pd, t_0)$  are constructed as described in Section IIC. For a generic quasiclassical coarse graining this conditional entropy should increase towards the future and decrease towards the past.

If present data contains a record of a particular alternative  $\alpha_0$ , then the conditional probability for it is unity and the conditional probability is zero for all other alternatives in any decoherent set containing it. The entropy of such a set is then

$$S(\{\alpha\}, t|pd, t_0) = \log \text{Tr}[P_{\alpha_0}(t)] . \quad (5.3)$$

This is *independent of time* as a consequence of the unitary evolution of the  $P$ 's (2.2) and the cyclic property of the trace. That is consistent with the second law. The second law thus does not rule out particular near certain predictions or retrodictions. For example, the entropy of a set of alternatives describing the positions of the earth and the moon to the accuracies current in celestial mechanics increases hardly at all in the near term. That is because their motion is essentially deterministic. A table of future eclipses is an example of a future record of such alternatives.<sup>10</sup>

The question, however, is not whether individual records are consistent with the second law, but whether present data contains more near certain retrodictions than near certain predictions. The second law implies that it does. Pick a set of quasiclassical alternatives  $\{P_\alpha(t)\}$ . Their entropy is generically greater at times after the time of present data than it is for times before. Therefore, generically the probabilities  $p(\alpha, t)$  are more distributed (less concentrated on one certain alternative) when  $t$  is to the future of the time of present data than when it is to the past of it. That means a given quasiclassical coarse graining is more likely to contain a near certain retrodiction of the past than a near certain prediction of the future.

This is in accord with the intuitive understanding that many records originate in irreversible processes. An impact crater on the moon, an ancient fission track in mica, or the arrangement of ink on this page are all examples of records in which the entropy rose significantly when they were created. Records created by irreversible processes must necessarily be records of the past to be consistent with the second law.

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<sup>10</sup> Note that the effort to compute the table does not enter the argument, nor should it. The present records of positions of the earth-moon system are correlated with their future positions whether or not these records are the input to explicit computation. From the point of view of entropy, computation is tantamount to copying and does not necessarily increase the entropy [20] unless outputs of a reversible computation are erased in accord with the Landauer's principle [21]. Computation is typically required in the process of prediction to generate *useful* records of future alternatives. In the computation of a table of eclipses an account of complex astronomical observations is converted to a more useful table of dates in the amended Gregorian calendar, both equally being records of the future astronomical events.

## VI. THE UTILITY OF HISTORY

We reconstruct the past to help predict the future. Certain regularities in the universe that can be inferred from present data can be understood as arising from historical events. This understanding is useful for predicting future regularities and that is the utility of history. In this section we shall amplify on this theme. We shall briefly discuss the nature of regularities, the process of historical explanation, quantitative measures of explanation, the role of theoretical input to that process, why explanations are mostly in the past, and the implications of many quantum pasts for historical explanation in quantum mechanics.

### A. The Origin of Regularities

Science is concerned with identifying regularities in the universe and exploiting these in the process of prediction. The laws which govern the regularities exhibited by all physical systems — without exception, without qualification, and without approximation — are the fundamental laws of physics. The present theoretical viewpoint is that there are two fundamental laws — a unified theory of the dynamical interactions, perhaps heterotic superstring theory or a generalization of it, and a theory of the initial quantum state of the universe.

Beyond the universal laws, science, including physics, aims at identifying and exploiting the regularities of specific physical systems. To give just a few examples: physics is concerned with the regularities of atoms, stars, and fluid flows; chemistry with the regularities of specific molecules; geology with the regularities of a specific planet; and the social sciences with the regularities exhibited by human beings — both collectively and individually. Specific classes of systems exhibit many more regularities than those implied by the fundamental laws of physics. What is the origin of the regularities peculiar to specific subsystems of the universe? Evidently they do not arise from the fundamental laws, as these govern only the regularities exhibited by all systems.

Every prediction in science can be regarded as a conditional probability in quantum cosmology. The regularities of specific systems can therefore only arise from the conditions entering the probabilities for these regularities. In some cases merely specifying the system is enough. To predict the emission spectrum of an isolated atom it is sufficient to specify that the atom is isolated in a certain excited state. For many other systems, however, the regularities arise from processes and events that have occurred over the course of the system's history<sup>11</sup> — some deterministic, some accidental. That is certainly the case for the regularities exhibited by the geology of the earth or by particular biological species that are the products of billions of years of chance mutation and natural selection. Even physics, however, is concerned with regularities which arise from historical events. The large scale distribution of galaxies in the visible universe is most succinctly understood as the outcome of some 15 billion years of evolution of primordial fluctuations by the action of gravitational attraction. This historical understanding from present data can be used to predict regularities in more detailed data yet to be obtained. For example, by reconstructing

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<sup>11</sup>See, *e.g.* [22] for a more detailed discussion.

the past history of the elements, we can predict the abundances in meteorites and stars yet to be observed. In the following we describe this process of historical explanation in quantum mechanics.

## B. Historical Explanation

From present data that includes most texts giving 55BC as the date of the Roman invasion of Britain, we may infer that Caesar did invade Britain in 55BC. That event, together with inferences from present data on the validity of the texts, is a *historical explanation* of the regularity in present data that most discussions of the Roman invasion give 55BC as its date. From this explanation one can predict that texts yet to be discovered will also give the date 55BC when describing the Roman invasion of Britain. (Of course, both inference and prediction are probabilistic; there is a probability that individual texts are forgeries, or contain mistakes, or that the ink on their pages made a quantum mechanical transition from a configuration spelling a different date.) The present coherence among texts on the date of the Roman invasion is an example of a “frozen accident” in the evocative terminology of Gell-Mann [22] — “chance events of which the particular outcomes have a multiplicity of long term consequences all related by their common ancestry”.

In principle the same prediction could be made from the present data itself — from the physical description of the texts and the configuration of ink molecules on their pages. However, it is evidently much easier to start from the event in the past. The reason is that present data contains much information that is irrelevant for this particular future prediction. It contains the specifications of the texts — their location, content, typeface language, number of pages, authors, *etc. etc.*, — many details to which the prediction of 55BC in future texts is not very sensitive. Of course, all that data is relevant for other predictions, those referring to the subsequent evolution of the texts for instance. But the reconstruction of past history permits the focus on the *essential* features of the regularity that can be inferred from present data for the prediction of the character of future texts.

The above discussion can be given both more generally and more concretely. Let  $P_{pd(t_0)}$  be the projection on present data held at time  $t_0$ , and let  $\{P_\alpha(t_0)\}$ ,  $\alpha = 1, 2, \dots$  be a set of alternatives in which  $P_{\alpha_0}(t_0)$  is a coarser projection than  $P_{pd(t_0)}(t_0)$  describing a feature, say a regularity, in present data. Histories

$$C_{\beta_0} = P_{\beta_{1,0}}^1(\tau_1) \cdots P_{\beta_{n,0}}^n(\tau_n) \quad (6.1)$$

are inferences from present data if

$$p(\beta_0|pd) \approx 1. \quad (6.2)$$

(In this section we will suppress the time labels of alternatives for compactness.) If among the possible inferences from present data, we can find one  $\beta_0$  for which

$$p(\alpha_0|\beta_0) \approx 1, \quad (6.3)$$

then we say that  $\beta_0$  is an explanation of  $\alpha_0$ .

In the history of the Roman invasion of Britain,  $pd$  contain all the details of present texts,  $\alpha_0$  is the coarse grained alternative that most texts report 55BC for the invasion,

and  $\beta_0$  is the chain of events in 55BC describing the invasion. In explaining the primordial abundance of the elements,  $pd$  includes the details of telescopic observations of the spectra from old, metal-poor stars,  $\alpha_0$  is the alternative that these all show approximately 75% H and 25% He, and  $\beta_0$  is the history of big bang nucleosynthesis in the early universe.

To consider probabilities such as (6.2) and (6.3) at all, the alternatives  $\beta_0$  and  $\alpha_0$  must be members of decohering sets of histories. As throughout this section, we assume the decoherence of the relevant sets of histories. We generally do not need to mention the set because the probabilities of specific alternatives such as  $\beta_0$  and  $\alpha_0$  are the same in all decoherent sets, being given by (2.4).

There will generally be many possible explanations of a given alternative  $\alpha_0$ . To give a trivial example, merely repeating present data or evolving it by the Schrödinger equation (2.2) to a different time would give a  $\beta_0$  satisfying (6.2) and (6.3). However, it is not explanation in general which is of interest, but rather explanation which is simpler than present data.

Quantitative measures of the simplicity of explanations depend on the context that is assumed in comparing them. The simplest measure is the information content (AIC) of the histories  $\beta_0$  satisfying (6.2) and (6.3). The smaller the AIC, the simpler the explanation. The notion of AIC was defined some thirty years ago by Kolmogorov, Chaitin, and Solomonoff [23] (all working independently) and has been employed in the definition of augmented thermodynamic entropy [24], the effective complexity of physical systems [25], and measures of classicality [12]. For a string of bits  $s$  and a particular universal computer  $U$ , the AIC of  $s$ , written  $K_U(s)$ , is the length of the shortest program that will cause  $U$  to print the string and then halt. A history like  $\beta_0$ , eq.(6.1), may be described by giving a description of its string of projection operators and their times. A description of a projection operator entails describing the operator whose eigenvalues are projected upon in terms of fundamental fields and specifying the range of these eigenvalues. If  $s$  is a description of such a history then the AIC of the string can be regarded as the AIC of the history. There will generally be many physically equivalent descriptions of a history in terms of fields [13]. For example, physically equivalent explanations are obtained by choosing different times for the operators in (6.1) and describing the operators by fields at that time. However, the AIC's of these equivalent histories may differ greatly. We therefore take the AIC of the history,  $K_U(\gamma)$  to be the minimum over all such descriptions.<sup>12</sup> The AIC,  $K_U(\beta_0)$ , of an explanation  $\beta_0$  then provides a quantitative way of distinguishing between different explanations by their relative simplicities. The  $\beta_0$  with the smallest AIC is the simplest explanation. Since  $\alpha_0$  trivially satisfies (6.2) and (6.3) it is an explanation of itself, and the simplest explanation of  $\alpha_0$  will have an AIC lower than that of  $\alpha_0$  itself.

Not every explanation, simple or not, is useful for prediction. The probabilities  $p(\gamma|pd)$  of an arbitrary set of alternatives  $\{P_\gamma^1(t_1)\}$  need be neither more nor less distributed than the probabilities  $p(\gamma|\beta_0)$  conditioned on an explanation of part of that data. However, if the explanation  $\beta_0$  captures the essential features of present data that are relevant for the alternatives  $\{P_\gamma^1(t_1)\}$ , then their prediction may be simplified. That will be the case when

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<sup>12</sup> More accurately, the AIC is defined on the physical equivalence class of descriptions of  $\alpha_0$  and  $\beta_0$  [13].



the probabilities of the  $\{P_\gamma^1(t_1)\}$  are independent of the part of present data not explained by  $\beta_0$  in the sense that

$$p(\gamma|pd, \beta_0) = p(\gamma|\beta_0) . \quad (6.4)$$

For such alternatives, the predictions from  $\beta_0$  agree with those from present data

$$p(\gamma|pd) = p(\gamma|\beta_0) , \quad (6.5)$$

because of

$$p(\gamma|pd) = \sum_\beta p(\gamma|pd, \beta)p(\beta|pd) \quad (6.6)$$

and eqs (6.2) and (6.4). When the explanation  $\beta_0$  is a simpler input to the prediction of the  $\{P_\gamma^1(t_1)\}$  than all of present data, the prediction of the future is simplified by historical explanation.

The above discussion does not assume that explanations must always be in the past, and indeed they need not be. The present position and velocity of the earth in its orbit around the sun might just as well be explained by some future position and velocity towards which it is heading as by any past position and velocity from which it came. However, the second law argument of the Section III implies that many explanations will be found in the past. Eq (6.2) shows that present data contains a record of the explanation  $\beta_0$ , and most, but not all, records are of the past because of the second law. Eq (6.3) shows that  $\beta_0$  is a record of the alternative  $\alpha_0$  to its *future*. However,  $\alpha_0$  is typically only a small part of present data — a coarse graining of it. It is more consistent with the general tendency of entropy to increase to have the records arranged in this way rather than the opposite time order. That is why the past is the most promising place to look for explanations.

There are no past explanations without theoretical input beyond present data. The explanation of the primordial abundance of the elements by big bang nucleosynthesis entails a number of assumptions about the nature of the early universe — its approximate homogeneity and isotropy, for instance. Those theoretical assumptions may be most properly understood as hypotheses about the universe's initial condition. Indeed we know from the discussion in Section IIIA that a theory of the initial condition is required for any retrodiction from present data in quantum mechanics.

As we have mentioned earlier, there will generally be many explanations — many  $\beta_0$  satisfying eqs (6.2) and (6.3) — of a given part of present data. These will differ in their simplicity and their utility for future prediction. There is no requirement that these different explanations be mutually compatible. They could belong to different quantum pasts in the sense of Section IIIB. With its many quantum pasts, the quantum theory of closed systems has more possibilities for historical explanation than it would if the theory were somehow restricted to a single decoherent set of histories. Mere decoherence in the presence of present data and the initial condition are all that is required to have a set of histories defining a quantum past. But to be useful for historical explanation, the probabilities for records and explanations in (6.2) and (6.3) must be near unity. The approximate determinism of the usual quasiclassical realm makes it more likely that these conditions will be satisfied in the usual quasiclassical past for usual quasiclassical present data than in other possible pasts. That is why the usual quasiclassical past is the most promising place to look for historical explanation. But we should not ignore the possibility of explanation through the many other pasts presented to us by quantum mechanics.

## VII. WHAT ABOUT US?

### A. Present, Past, and Future

“Observers” or “measurements” have no fundamental role in the quantum mechanics of closed systems. Of course, “observers” can be described as special physical systems (typically complex ones) within the universe, and “measurements” can be described as special interactions between subsystems of the universe. “Observers” and “measurements” are of special interest to us because that’s what we are and that’s how we learn about the universe. How then do information gathering and utilizing systems (IGUSes) such as ourselves distinguish the present moment, its future, and its past? The author is not equipped to speculate on the detailed operation of the human brain. However, it seems likely that mechanical systems such as computers could be constructed and programmed to draw similar distinctions between past, present, and future. We proceed by discussing how such a system could operate. We restrict attention to IGUSes whose description and observations are coarse-grainings of the usual quasiclassical realm. The discussion in this section is necessarily more suppositional than that in the rest of the paper, and the reader should take note of that. Nevertheless, it concerns questions that can be seriously addressed in science, even if now only by conjecture and not calculation, as here.

An IGUS holds a set of records. As time changes (in either direction!) this set of records also changes. New records appear and others disappear or are altered. The records can therefore be approximately time ordered. How accurate the records are at any time, how long and faithfully they persist, and how accurately they can be time ordered depends on the particular IGUS. Everyday experience shows considerable variation in these qualities from IGUS to IGUS.

If the records of an IGUS are examined at any one time we expect some to be correlated with external events near that time. That is the present input, and these records constitute the IGUS’s notion of the present. To the extent that the IGUS’s records are acquired in irreversible processes, we expect most of the rest to be records of the *past* as a consequence of the second law as discussed in Section III. That is why we say we remember the past and not the future.<sup>13</sup> However, an IGUS is not prohibited from possessing a record of the future. It might have memorized a table of future eclipses. It might have correctly conjectured the motion of a fly or the movement of tomorrow’s stock market. An IGUS must predict to function, which means creating useful approximate records of the future. As the above examples illustrate, such records of the future are typically distinguishable from records of the past as the outcomes of *computation* involving other records as input, mostly records of the past. That is how we can say that we predict the future but remember the past.

Thus, like all other time asymmetries, the subjective distinction between past, present, and future is traceable to properties of the universe’s initial condition — in this case its low entropy in terms of usual quasiclassical variables. All other aspects of the phenomena loosely called the “psychological arrow of time” presumably have similar explanations. For

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<sup>13</sup>The author is not alone in connecting the “psychological arrow of time” to the thermodynamic arrow of time, see *e.g.* [17,26].

example, we have the impression that we can control the future, but the past is over and done with. What is meant by “control” is the amplification of a pattern of weak electrical activity in the brain to large irreversible effects correlated with that pattern. The second law means that such effects are much more likely to be in the future of the pattern of activity than to its past.

### B. Which Past Do We Remember?

Which of the many possible quantum pasts do we remember? As IGUSes we are adapted to observe and record alternative values of quasiclassical operators. It is no surprise that we individually remember quasiclassical pasts if our records are almost entirely of quasiclassical alternatives. Further, these individual quasiclassical pasts are all coarse grainings of the usual quasiclassical realm which is why they are in approximate agreement. However, at a deeper level, we may ask: “What is the reason for our focus on quasiclassical alternatives?” Our particular properties as physical systems in the universe can only be understood in the context of our evolution within it. The usual quasiclassical realm of decoherent histories has a high level of predictability in time. Such properties make it plausible that IGUSes described in terms of usual quasiclassical variables evolved to make use of coarse grainings of the usual quasiclassical realm because its relative predictability makes it adaptive to do so in comparison with more quantum mechanical alternatives they might have used. Probabilities relevant for that adaptation are in principle computable from quantum mechanics although well beyond our ability to do so at present.<sup>14</sup>

### C. The Marvelous Moment “Now”

To function, an IGUS employs a self-referential, evolving model or schemata of itself and its external environment [22]. Such a model necessarily distinguishes records of present events from ones in the future and past as described above and, in the case of human beings, devotes a significant amount of conscious focus to records of the most recent input. The author knows of no physical reason why a device could not be constructed which gives equal conscious focus to records of an hour ago as well as of the present. Plausibly it has been adaptive to concentrate on the present and near past. However, our preponderance of attention to the present does not mean that “now” is somehow distinguished in physics. Neither is it evidence that for the more radical but problematical suggestion that physics can be formulated entirely at one marvelous moment “now” [27]. To give just one reason, the moment “now” is only approximately defined to the accuracy of some processing time in the brain and certainly not to the accuracy of Planck time,  $10^{-43}$ sec, on which spacetime can be defined.<sup>15</sup> “Now” is a moment in a schemata of an IGUS, not a preferred surface in the four-dimensional spacetime continuum.

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<sup>14</sup>For further discussion see [13].

<sup>15</sup>For more reasons see, *e.g.* [6], Section V.1.2.

## VIII. CONCLUSION

The quantum mechanical past is a country of very different appearance from the past of classical physics. Present data, no matter how refined, are not enough for any retrodictions without being augmented by a theory of the initial condition. Even then, there are many mutually incompatible pasts, each a possible source of historical explanation. As new data is acquired none of these possible quantum pasts need continue to be retrodictions — a quantum past is not necessarily permanent. The familiar properties of uniqueness, permanence, and retrodictibility from present data alone, that are characteristics of the classical past emerge in quantum theory, not generally, but as approximations appropriate to particular coarse-grainings and particular initial conditions such as that for our universe.

The multiplicity of mutually incompatible sets of alternative decoherent histories that can be employed for prediction and retrodiction in the quantum mechanics of closed systems has motivated some to search for additional principles that would restrict the available sets in some way [28,29] and thereby necessarily recover properties of the quasiclassical past that are not available generally in quantum theory. Sum-over-histories formulations [30], strong decoherence [13], and ordered consistency [31] can all be seen as efforts in this direction. With such restrictions the properties of quantum past could be different and perhaps closer to those of classical physics. It is important to stress, however, that, even without such additional principles, the quantum theory of closed systems is logically consistent, consistent with experiment as far as is known, and applicable for prediction and retrodiction to the most general physical systems.

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